Math 143 Sample Exam 1

Question 1 For each of the following sequences $\{a_n\}$, decide whether it converges or diverges and circle the appropriate word. If the sequence converges, compute the limit of the sequence and write the limit of the sequence in the blank. (Show all work.)

a) $a_n = \dfrac{\sqrt{2n^2+10n-1}}{4-6n}$ **Converges** **Diverges** **Limit=** ______________

b) $a_n = \pi^{(3-\cos(3/n))}$ **Converges** **Diverges** **Limit=** ______________

c) $a_n = \sin\left(\dfrac{\pi}{2} + (-1)^n\pi\right)$ **Converges** **Diverges** **Limit=** ______________

d) $a_n = \dfrac{(n+2)!}{(n-1)!} + \left(\dfrac{n+1}{n-1}\right)^n$ **Converges** **Diverges** **Limit=** ______________

Question 2 For each of the following series decide whether the series converges or diverges and circle the appropriate word. Write the name of the test used to decide in the blank. (Show all work.)

a) $\displaystyle\sum_{n=1}^{\infty} \left(2 - e^{(-1/n)}\right)$ **Converges** **Diverges** **Test Used=**______________

b) $\displaystyle\sum_{n=1}^{\infty} \frac{4\sqrt[3]{n^5}}{n^2}$ **Converges** **Diverges** **Test Used=**______________

c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{3n}}{10^n}$ **Converges** **Diverges** **Test Used=**______________

d) $\displaystyle\sum_{n=0}^{\infty} \frac{n}{\sqrt[3]{n^2+1}}$ **Converges** **Diverges** **Test Used=**______________

e) $\displaystyle\sum_{n=0}^{\infty} \frac{5n-3}{n^2-2n+43}$ **Converges** **Diverges** **Test Used=**______________

f) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(n!)^2\, 3^n}{(2n+1)!}$ **Converges** **Diverges** **Test Used=**______________

Question 3 Compute the sum of the following infinite series:

a) $\displaystyle\sum_{n=3}^{\infty} \frac{(-3)^{n-2}}{7^{n+1}}$

b) $\displaystyle\sum_{n=2}^{\infty} \frac{4}{n(n+2)}$

Question 4 Does the following series converge conditionally, absolutely or diverge:

$$\sum_{n=1}^{\infty} (-1)^n \frac{(\ln n)^{2002} + \sin^2 n}{\sqrt[5]{n^6+11}}$$

Question 5 Find the interval of convergence of the power series:

$$\sum_{n=0}^{\infty} (-1)^n \frac{(3x+1)^n}{4^{2n}(n+1)}$$

Don't forget to check the endpoints!

Question 6 [20 points] Evaluate the indefinite integral $\displaystyle\int x\cos(3x^3)\,dx$ as a power series. You will need to know that $\cos x = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$.